Cost minimization and interpretation of the lagrange multiplier

A firm produces two items, A and B, through a joint production system, with the cost function: $C(q_1, q_2) = q_1^2 + q_2(2q_2 - q_1)$. Where q_1 and q_2 are the quantities of A and B, respectively.

1. If the total quantity of both products should be eight units, determine the quantities of each product that minimize the cost.

2. Without solving the problem again, estimate what the minimum cost would be if the firm has an availability of two additional units of product.

Solution

1. We set up the Lagrangian:

$$L = q_1^2 + 2q_2^2 - q_2 q_1 + \lambda(8 - q_1 - q_2)$$

We set up the first-order conditions:

$$L'_{q_1} = 2q_1 - q_2 - \lambda = 0$$

$$L'_{q_2} = 4q_2 - q_1 - \lambda = 0$$

$$L'_{\lambda} = 8 - q_1 - q_2 = 0$$

We solve for and equate λ:

$$2q_1 - q_2 = 4q_2 - q_1$$

$$3q_1 = 5q_2$$

$$q_1 = \frac{5}{3}q_2$$

Insert into the third condition:

$$8 - \frac{5}{3}q_2 - q_2 = 0$$

$$8 = \frac{8}{3}q_2$$

$$q_2 = 3$$

$$q_1 = 5$$

2. We calculate the value of $\lambda = 2 * 5 - 3 = 7$ Multiplying $7 * 2 = 14$ would be the additional cost of two units. The previous cost was:

$$C = 5^2 + 3(2 \cdot 3 - 5) = 28$$

The total is $28 + 14 = 42$.